

Inelastic Final State Interactions in $B \rightarrow PP$ Decays

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A method parametrizing all inelastic final state interactions (FSI) in $B \rightarrow PP$ decays is presented (P - pseudoscalar meson). The method explicitly shows how rescattering leads to the replacement of the short-distance amplitudes with the effective quark diagram amplitudes, and how it affects the extraction of the unitarity triangle angle γ from the data. It is furthermore pointed out that the size of FSI effects cannot be determined from $B_d^0 \rightarrow K^+ K^-$ decays in a satisfactory way. The case of $SU(3)$ -violating FSI is also discussed. When fits to the branching ratios of all $B \rightarrow PP$ decays are performed with all inelastic FSI included, the extracted value of γ is shifted down by some $20^\circ - 30^\circ$ when compared to the no-FSI analyses, and becomes consistent with the Standard Model value of $65^\circ \pm 7^\circ$.

1. Introduction

One of the objectives of contemporary studies of B meson decays is to check whether their description provided by the Standard Model (SM) is correct. Should the values of the SM parameters, extracted in various ways, turned out to be inconsistent, we might conclude that some kind of new physics is needed. Since new physics is expected to enter through loop diagrams, it should appear in penguin amplitudes.

At the same time one has to keep in mind that diagrams of penguin topology can be also generated by ordinary final state interactions (FSI). Thus, it is important that all effects of FSI are subtracted before any claim as to the presence of new physics is made. However, controlling rescattering effects in B decays is non-trivial as FSI may be highly inelastic [1]. Indeed, it follows from our knowledge of high energy scattering that when two pseudoscalar mesons PP collide at B -meson energy, a many-body state is generally produced. Therefore, in $B \rightarrow PP$ decays one may expect contributions not only from the standard $B \rightarrow P_1 P_2$ quark-level transitions (corrected for the elastic $B \rightarrow P_1 P_2 \rightarrow P_1 P_2$, and quasi-elastic $B \rightarrow P_1' P_2' \rightarrow P_1 P_2$ transitions), but

also from the inelastic $B \rightarrow M_1 M_2 \dots M_n \rightarrow P_1 P_2$ processes with many mesons in the intermediate state. Taking into account all of the latter FSI effects is possible only if substantial simplifications in their description are made.

2. Simplified description of FSI

Since contributions from inelastic FSI are incalculable, the only feasible way to consider them is to express somehow their effects in terms of a small number of effective parameters. The issue of how to do that was addressed in [2,3], where several essential simplifications, listed below, were made.

First, it was assumed that the FSI effects lead to an only small correction to the standard description in terms of short-distance (SD) amplitudes. The FSI-corrected amplitudes \mathbf{W} (a set of all relevant amplitudes $B \rightarrow \text{hadrons}$) could therefore be written as

$$\mathbf{W} = \mathbf{w} + \mathbf{R}\mathbf{w}, \quad (1)$$

where \mathbf{w} denotes all $B \rightarrow \text{hadrons}$ SD amplitudes and \mathbf{R} is the rescattering matrix.

Second, it was assumed that FSI is $SU(3)_F$ symmetric (with $SU(3)_F$ breaking considered later). In this way, rescattering effects in all $B^\pm, B_{d,s}^0, \bar{B}_{d,s}^0 \rightarrow PP$ decays are related.

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Third, an essential simplification was made in the treatment of many-body intermediate states which may occur in $B \rightarrow P_1 P_2$ transitions. As the original weak decay leads initially to $q\bar{q}$ or $qq\bar{q}\bar{q}$ states which evolve into many-body states only later, e.g.:

$$B \rightarrow qq\bar{q}\bar{q} \rightarrow M_1 M_2 \dots M_n \rightarrow P_1 P_2, \quad (2)$$

it is natural to include the second transition above ($qq\bar{q}\bar{q} \equiv M'_1 M'_2 \rightarrow M_1 M_2 \dots M_n$) into the definition of FSI. Thus, as the intermediate states only two-body states $M'_1 M'_2$ may be taken (the $q\bar{q}$ state may also be considered as decaying to $M_1 M_2 \dots M_n$ through the $M'_1 M'_2$ stage).

Fourth, a way to sum over all intermediate $M_1 M_2$ states was proposed (hereafter we suppress the primes in $M'_1 M'_2$). Consider e.g. a tree amplitude $T_{M_1 M_2}$ for the production of a general two-body intermediate state $M_1 M_2$. Without losing generality one can always write:

$$T_{M_1 M_2} = \eta_{M_1 M_2}^T T_{P_1 P_2}, \quad (3)$$

i.e. express the amplitude $T_{M_1 M_2}$ in terms of the SD tree amplitude $T_{P_1 P_2}$. Similarly one can always write for the penguin amplitude:

$$P_{M_1 M_2} = \eta_{M_1 M_2}^P P_{P_1 P_2}, \quad (4)$$

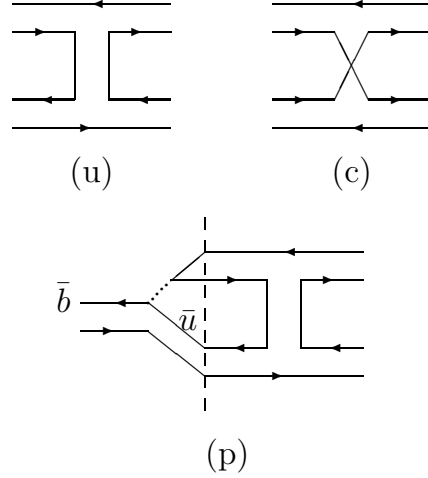
with analogous expressions for other diagram types. Since FSI effects constitute a correction, only the dominant SD amplitudes (i.e. T, P and the color-suppressed C amplitude in $\Delta S = 0$ decays) need to be taken into account. The essential simplification consists here in assuming that:

$$\eta_{M_1 M_2}^T = \eta_{M_1 M_2}^P = \dots = \eta_{M_1 M_2}. \quad (5)$$

The above assumption helps in reducing the number of effective FSI parameters. Further drastic reduction in their number is achieved via the summation over all intermediate states $M_1 M_2$. Consider for example the contribution from tree amplitudes $T_{M_1 M_2}$ with a part of FSI in which no flavor quantum numbers are exchanged in the t -channel of the rescattering amplitude (Pomeron exchange). Denoting the $M_1 M_2 \rightarrow P_1 P_2$ amplitude by $f_{M_1 M_2}$ one can then write

$$\sum_{M_1 M_2} T_{M_1 M_2} f_{M_1 M_2} = R_f T_{P_1 P_2}, \quad (6)$$

Figure 1. Quark line diagrams: (u) uncrossed, (c) crossed, (p) penguin generation.



where $R_f \equiv \sum_{M_1 M_2} \eta_{M_1 M_2} f_{M_1 M_2}$ was introduced as an effective parameter. If the part of FSI with the topology of e.g. Fig.1(u) is taken into account one similarly obtains:

$$\sum_{M_1 M_2} T_{M_1 M_2} g_{M_1 M_2} = R_g P_{P_1 P_2}, \quad (7)$$

with appropriately defined R_g , i.e. an effective penguin diagram is created (Fig. 1(p)).

Fifth, Zweig rule was assumed. This means that only two types of (flavor transfer) FSI diagrams are possible as shown in Fig.1(u) (uncrossed), and Fig.1(c) (crossed). For FSI of the uncrossed type two $SU(3)_F$ forms are possible:

$$Tr(\{M_1^+, M_2^+\}\{P_1, P_2\})u_+(M_1 M_2), \quad (8)$$

$$-Tr([M_1^+, M_2^+]\{P_1, P_2\})u_-(M_1 M_2), \quad (9)$$

where $u_{\pm}(M_1 M_2)$ are parameters describing the strength of the relevant transitions. Bose symmetry requires that the final PP state is described by a symmetric form $\{P_1, P_2\}$. Charge conjugation invariance of strong interactions requires that the product of C -parities of mesons $^2 M_1 M_2$ is positive for (8) and negative for (9), respectively:

$$C_{M_1 M_2} = +C_{P_1 P_2} = +1, \quad (10)$$

$$C_{M_1 M_2} = -C_{P_1 P_2} = -1. \quad (11)$$

²C-parities of whole $SU(3)_F$ meson multiplets are defined to be the C -parities of their neutral members

For the FSI of the crossed type only one $SU(3)_F$ form, symmetric under $P_1 \leftrightarrow P_2$, can be written:

$$Tr(M_1^+ P_1 M_2^+ P_2 + M_1^+ P_2 M_2^+ P_1) c(M_1 M_2). \quad (12)$$

As it was shown in [2,3], when summation over $M_1 M_2$ is performed all inelastic FSI effects are ultimately reduced to the appearance of *three* effective FSI parameters (analogues of R_f, R_g) only:

$$\begin{aligned} u_R &\equiv u_+ + u_- \\ d_R &\equiv (u_+ - u_-)/2 \\ c_R &\equiv c, \end{aligned} \quad (13)$$

(u_+ involves now sums of terms including $u_+(M_1 M_2)$, etc.). If only $P'_1 P'_2$ intermediate states are admitted, it follows that $u_- = 0$ and only two parameters remain.

3. Effective quark diagrams

When the sums over all types of $\{M_1, M_2\}$ states and over all types of $[M_1, M_2]$ states are performed, one obtains expressions for the FSI-corrected $B \rightarrow PP$ amplitudes $\mathbf{W} = \mathbf{w} + \mathbf{R}(u_R, d_R, c_R)\mathbf{w}$. For $SU(3)_F$ -symmetric FSI the obtained expressions are identical in form to the SD expressions, but with redefined amplitudes [2]. Specifically, taking selected $\Delta S = 0$ amplitudes as an example, one obtains:

$$\begin{aligned} W(B^+ \rightarrow \pi^+ \pi^0) &= -(\tilde{T} + \tilde{C})/\sqrt{2} \\ W(B_d^0 \rightarrow \pi^+ \pi^-) &= -(\tilde{T} + \tilde{P}) - (\tilde{E} + \tilde{P}A) \\ W(B_d^0 \rightarrow K^+ K^-) &= \tilde{E} + \tilde{P}A \\ W(B_d^0 \rightarrow K^0 \bar{K}^0) &= -\tilde{P} - \tilde{P}A, \end{aligned} \quad (14)$$

where

$$\begin{aligned} \tilde{T} &= T + 2c_R C \\ \tilde{C} &= C + 2c_R T \\ \tilde{P} &= P + u_R(T + 3P) \\ \tilde{A} &= 2d_R C \\ \tilde{E} &= 2d_R T \\ \tilde{P}A &= 4d_R P. \end{aligned} \quad (15)$$

Analogous formulas hold for $|\Delta S| = 1$ amplitudes T', P' , etc. When explicit weak phases (with $\lambda_k^{(d)} = V_{kd} V_{kb}^*$, V being the CKM matrix)

are introduced through $T \equiv \lambda_u^{(d)} t$, and the top-dominated penguin $P \equiv \lambda_t^{(d)} P_t$ is assumed, one finds that the redefined penguin \tilde{P} is of the form

$$\begin{aligned} \tilde{P} &= \lambda_t^{(d)} P_t (1 + 3u_R) + \lambda_u^{(d)} t u_R \\ &= \lambda_t^{(d)} \tilde{P}_t + \lambda_u^{(d)} \tilde{P}_u, \end{aligned} \quad (16)$$

i.e. the "top" penguin gets rescaled, and an "up" penguin appears.

Rescattering induces also the appearance of annihilation (\tilde{A}), exchange (\tilde{E}), and penguin annihilation ($\tilde{P}A$), all proportional to d_R . In particular, note that the amplitude for the $B_d^0 \rightarrow K^+ K^-$ decay is proportional to d_R . Thus the branching ratio for this decay does not yield any information on the remaining two FSI parameters (u_R, c_R) (see e.g. [4]). One should think of $u_R = u_+ + u_-$ as originating from one superposition of contributions from intermediate $C_{M_1 M_2} = +1$ and $C_{M_1 M_2} = -1$ states, with $d_R \propto u_+ - u_-$ being due to the other superposition. Thus, the two contribution may cancel in d_R , while adding in u_R . Only if we knew that pseudoscalar mesons alone contribute in the intermediate states, would the measured size of $B_d^0 \rightarrow K^+ K^-$ indeed tell us about the size of the "up" penguin \tilde{P}_u .

The third FSI parameter, c_R , redefines the tree and color-suppressed diagrams according to the formula:

$$\frac{\tilde{C}}{\tilde{T}} = \frac{\frac{C}{T} + 2c_R}{1 + 2c_R \frac{C}{T}}, \quad (17)$$

which shows that an originally small size of C/T could be substantially affected by FSI of the crossed type.

4. $SU(3)_F$ breaking

Since in the real world $SU(3)_F$ is broken, for the purpose of fitting the data on $B \rightarrow PP$ decays it is appropriate to break $SU(3)_F$ both in the elastic, as well as in the quasi-elastic and inelastic contributions. Treatment of $SU(3)$ breaking in elastic FSI is straightforward: from total cross-section data on $\pi p \rightarrow \pi p$, $K p \rightarrow K p$, etc. one can extract the relevant $SU(3)_F$ -breaking couplings of Pomeron to mesons. Thus, elastic $SU(3)_F$ -breaking FSI effects in $B \rightarrow P_1 P_2$ are fully calculable (see [2,3]).

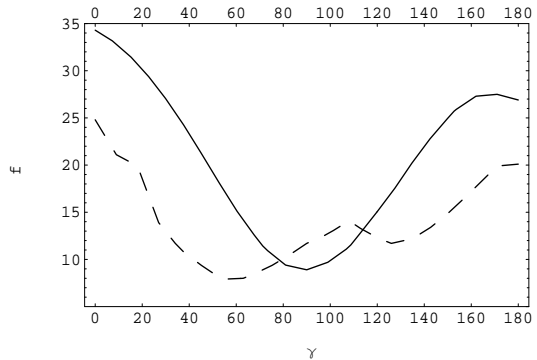
For the inelastic (and quasi-elastic) $M_1 M_2 \rightarrow P_1 P_2$ transitions one expects the annihilation (or exchange) of strange (anti)quarks to be suppressed when compared to analogous amplitudes with all quarks non-strange. Data on hadron-hadron collisions at B -mass energy indicate that the relevant suppression factor ϵ is much smaller than its $SU(3)_F$ -suggested value of 1. Setting $\epsilon \neq 1$ invalidates the use of $SU(3)_F$ formulas. In fact, for the $SU(3)_F$ breaking case the general FSI formulas do not permit quark diagrams to be redefined in a way analogous to Eqs.(15) and valid simultaneously for all amplitudes. Instead, one has to use the full form of relevant $SU(3)_F$ -breaking expressions given in [3].

It was suggested that an estimate of the size of rescattering may be obtained from a comparison of the branching ratios of U -spin-related decays $B^+ \rightarrow K^+ \bar{K}^0$ and $B^+ \rightarrow \pi^+ K^0$ [5]. In a world with $SU(3)_F$ -symmetric FSI this could be done since in these two decays the u -quark penguins generated from tree diagrams according to Fig.1(p) are of different relative magnitudes when compared with the dominant t -quark penguins. Indeed, using $P \approx \lambda P'$ and $T \approx T'/\lambda$ ($\lambda \approx 0.22$ being the Wolfenstein parameter) one finds that in $B^+ \rightarrow K^+ \bar{K}^0$ the ratio of the tree-generated penguin to the original penguin is $u_R T/P \approx T' u_R/P' \cdot 1/\lambda^2$, while for the $B^+ \rightarrow \pi^+ K^0$ decay the relevant ratio is simply $T' u_R/P'$. With $1/\lambda^2 \approx 20$, the FSI effects should be much more pronounced in $B^+ \rightarrow K^+ \bar{K}^0$. However, when $SU(3)_F$ breaking in FSI is taken into account the FSI effects in $B^+ \rightarrow K^+ \bar{K}^0$ become suppressed by a factor of ϵ (with the creation of a new $s\bar{s}$ pair being unlikely), and the overall difference between the size of rescattering effects in the two modes becomes proportional to ϵ/λ^2 , which may be much closer to 1 than $1/\lambda^2$.

5. Fits to $B \rightarrow PP$ branching ratios

In the actually performed fits to the $B \rightarrow PP$ branching ratios the following sixteen channels were considered: 3 $\pi\pi$ channels, 4 πK , 3 $K\bar{K}$, 2 ηK , 2 $\eta' K$, $\eta\pi^+$, and $\eta'\pi^+$. The fits consisted in minimizing the χ^2 function involving theoretical and experimental branching ratios \mathcal{B} and their

Figure 2. Fits with inelastic FSI: 1) dashed line - u_R free, $c_R = 0$; 2) solid line - $u_R = 0$, c_R free.



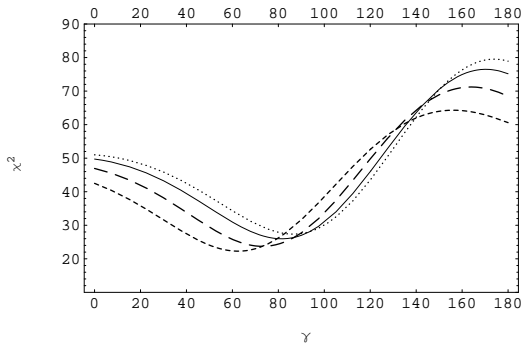
experimental errors $\Delta\mathcal{B}$:

$$\chi^2 = \sum_1^{16} \frac{|\mathcal{B}_i(exp) - \mathcal{B}_i(the)|^2}{|\Delta\mathcal{B}_i(exp)|^2}. \quad (18)$$

Four SD parameters were used: $|T|$, $P' = -|P'|$, the singlet penguin S' , and γ . The remaining amplitudes were related by $T' = \frac{V_{us}}{V_{ud}} \frac{f_K}{f_\pi} T$, and $P = -e^{-i\beta} \left| \frac{V_{td}}{V_{ts}} \right| P'$ (with $\beta = 24^\circ$), and by $C = \xi T$, $C' = T'(\xi - (1 + \xi)\delta_{EW}e^{-i\gamma})$, with $\xi = 0.17$ and including the dominant electroweak penguin with $\delta_{EW} = 0.65$. Prior to the inclusion of FSI, all amplitudes were assumed to have weak phases only.

Fits performed for the case of $SU(3)$ -breaking pure elastic FSI showed the latter to be negligible, and led to γ around 100° (as in the case with no FSI at all). In all fits involving free inelastic FSI parameters (in addition to $|T|$, P' , etc.), the value of d_R was set to 0, and maximal $SU(3)$ breaking was assumed ($\epsilon = 0$). Then, two simplified cases were studied first: 1) with vanishing c_R and free complex u_R , and 2) with vanishing u_R and real ic_R (this restriction takes care of direct-channel no-exotics condition for the crossed-type diagrams.) Results of the fits are shown in Fig.2. One can see that substantial shifts in the fitted value of γ , away from the no-FSI value of around 100° , are obtained. In a general fit performed with both u_R and c_R free, a very shallow global minimum was found close to the one obtained in case 1) above. Thus, the general fit permits val-

Figure 3. Fits for charming penguin contribution only: $P_c/P_t = 0.6, 0.4, 0.0, -0.6$ for short-dashed, long-dashed, solid, and dotted lines respectively



ues of γ in a broad range, including its standard model value $\gamma_{SD} \approx 65^\circ$ (for more details see [3]).

6. Charming penguins

One might question whether FSI can indeed shift the value of γ that much. However, shifts of similar size are obtained also when performing fits for charming penguins only, i.e. without inelastic rescattering involving intermediate states composed of light quarks. Namely, in [6] it was shown that one should expect the charming penguin contribution P_c to be comparable with P_t :

$$0.2 < \left| \frac{P_c - P_u}{P_t - P_u} \right| < 0.5. \quad (19)$$

Results of the fits (shown in Fig.3) performed with $P_c \neq 0$ and $P_u = 0$ for an updated set of branching ratios indicate [7] that for the charmed penguin of size expected in ([6]), the shift in γ may well be of the order of 20° degrees.

7. Conclusions

In standard short-distance approaches with built-in $SU(3)_F$ -symmetry the amplitudes for B-meson weak decays may be decomposed into several quark diagram amplitudes. When $SU(3)_F$ -symmetric final state interactions are taken into account, the resulting FSI-corrected amplitudes

must also be decomposable into several *effective* quark diagram amplitudes. In this talk the connection between the two sets of quark diagram amplitudes was presented for the appropriately simplified but still general case of inelastic FSI. Thus, an understanding of how FSI redefine original quark diagram amplitudes was reached.

In particular it was shown that all leading effects of $SU(3)_F$ -symmetric FSI may be parametrized by only three effective parameters. Experimental upper bound on the $B_d^0 \rightarrow K^+ K^-$ branching ratio limits the size of one of these parameters only. The remaining two parameters redefine the penguin amplitude and mix the tree and color-suppressed amplitudes.

From the fits to sixteen branching ratios of $B \rightarrow PP$ decays performed with no FSI taken into account it follows that such fits are still quite sensitive to the uncertainties present in the experimental data (with the fitted value of γ changing from around 100° to 80°). The fits performed with maximal $SU(3)_F$ breaking in inelastic FSI (not including charmed intermediate states) admit a strong shift in the extracted value of γ (within the range of $\gamma \in (60^\circ, 110^\circ)$), favoring the SM value of γ very weakly. Similar shifts and conclusions are obtained from the fits when only charming penguins are considered.

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